

**THE KAVERY ENGINEERING COLLEGE***(An Autonomous Institution)***B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, NOV/DEC 2025***Third Semester**Mechanical Engineering***MA3351 / UMAT24301 - Transforms and Partial Differential Equations***(Common to Biomedical Engineering and Civil Engineering)***Regulations – 2021 & 2024***Duration : 3 Hrs**Maximum : 100 Marks***PART – A (ANSWER ALL QUESTIONS) (Marks 10\*2=20)**

| <i>Q.No</i> | <i>Questions</i>                                                        | <i>BLT</i> | <i>CO</i> | <i>Marks</i> |
|-------------|-------------------------------------------------------------------------|------------|-----------|--------------|
| 1.          | Form the PDE of all spheres whose centers lie on the z-axis.            | L1         | 1         | 2            |
| 2.          | Solve $(D^2 + 3DD' + 2D'^2)z = 0$ .                                     | L3         | 1         | 2            |
| 3.          | State the convergence condition on Fourier series.                      | L1         | 2         | 2            |
| 4.          | What do you mean by Harmonic Analysis?                                  | L1         | 2         | 2            |
| 5.          | Classify the pde $4u_{xx} + 4u_{xy} + u_{yy} - 6u_x - 8u_y - 16u = 0$ . | L3         | 3         | 2            |
| 6.          | Write the possible solution of one dimensional wave equation.           | L1         | 3         | 2            |
| 7.          | Write any two applications of Fourier Transform.                        | L1         | 4         | 2            |
| 8.          | State convolution theorem of Fourier Transform.                         | L1         | 4         | 2            |
| 9.          | Find $z[n\cos(n\theta)]$ .                                              | L3         | 5         | 2            |
| 10.         | Find the inverse Z-transform of $\frac{1}{1-az^{-1}}$ .                 | L3         | 5         | 2            |

**PART – B (ANSWER ALL QUESTIONS) (Marks 5\*16 = 80)**

| <i>Q.No</i> | <i>Questions</i>                                                                                                                                              | <i>BLT</i> | <i>CO</i> | <i>Marks</i> |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-----------|--------------|
| 11.         | a i Find the singular integral of $z = px + qy + \sqrt{p^2 + q^2 + 1}$ .                                                                                      | L3         | 1         | 8            |
|             | ii Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(x^2 - y^2)$ .                                                                                                     | L3         | 1         | 8            |
|             | Or                                                                                                                                                            |            |           |              |
|             | b i Solve $(D^3 + D^2D' - DD'^2 - D'^3)Z = e^{2x+y} + \cos(x+y)$ .                                                                                            | L3         | 1         | 8            |
|             | ii Find the general solution of $(D^2 + DD' - 6D'^2)z = y\cos x$ .                                                                                            | L3         | 1         | 8            |
| 12.         | a i Expand $f(x) = x(2\pi - x)$ as a Fourier series in $(0, 2\pi)$ and hence deduce that the sum of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ . | L3         | 2         | 8            |
|             | ii Find the half range sine series of $f(x) = x \cos \pi x$ in $0 < x < 1$ .                                                                                  | L3         | 2         | 8            |
|             | Or                                                                                                                                                            |            |           |              |

- b i Find the complex form of Fourier series for  $f(x) = e^{-ax}$  in  $-l < x < l$ . L3 2 8
- ii Compute the first two harmonic of the  $f(x)$  from the following table: L3 2 8

| x      | 0   | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ | $2\pi$ |
|--------|-----|-----------------|------------------|-------|------------------|------------------|--------|
| $f(x)$ | 1.0 | 1.4             | 1.9              | 1.7   | 1.5              | 1.2              | 1.0    |

13. a A string is stretched and fastened at two points that are distance  $l$  apart. Motion is started by displacing the string into the form  $y = kx(l - x)$  from which it is released at time  $t = 0$ . Find the displacement of the string at any point at any time  $t$ . L3 3 16

Or

- b A bar 10 cm long insulated sides have its ends A and B kept at  $20^\circ\text{C}$  and  $40^\circ\text{C}$  respectively until steady state conditions prevail. The temperature at A is then suddenly raised to  $50^\circ\text{C}$  and at the same instant that at B is lowered to  $10^\circ\text{C}$ . Find the subsequent temperature at any point of the bar at any time. L3 3 16

14. a i Show that  $e^{-\frac{x^2}{2}}$  is self reciprocal under the fourier transform by finding the transform of  $e^{-a^2x^2}$ . L3 4 8

- ii Find the Fourier cosine transform of  $f(x) = \frac{e^{-ax} - e^{-bx}}{x}; x > 0$ . L3 4 8

Or

- b Find the Fourier transform of  $f(x) = \begin{cases} 1 - x^2 & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$  and hence show that  $\int_0^\infty \frac{\sin t - t \cos t}{t^2} dt = \frac{\pi}{15}$  and also  $\int_0^\infty \left( \frac{\sin t - t \cos t}{t^3} \right) \cos\left(\frac{s}{2}\right) dt = \frac{3\pi}{16}$ . L3 4 16

15. a i Find the z transform of  $\cos \frac{n\pi}{2}$  and  $\frac{1}{n(n+1)}$ . L3 5 8

- ii Find the inverse Z transform of  $\frac{z^2 + z}{(z-1)(z^2+1)}$  using partial fraction. L3 5 8

Or

- b i Using the convolution theorem solve  $z^{-1} \left[ \frac{z^2}{(z-3)(z-4)} \right]$ . L3 5 8

- ii Solve the equation  $y_{n+2} - 7y_{n+1} + 12y_n = 2^n$  given that  $y_0 = y_1 = 0$ . L3 5 8